$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$
$\hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$
$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$

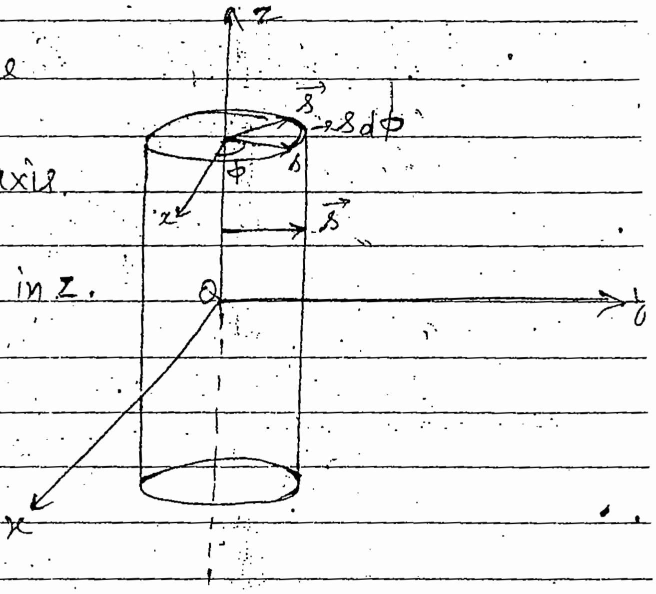
$\hat{x} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$
$\hat{y} = \sin\theta\sin\phi\,\hat{r} + \sin\phi\cos\theta\,\hat{\theta} + \cos\phi\,\hat{\phi}$
$\hat{z} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

Cylendrical Co-ordinate system :-

s, ϕ, z are mutually orthogonal to eachother.

$\vec{s}$ is radius vector of the cylender.
$\phi \rightarrow$ angle b/w $\vec{s}$ & x-axis.
$z = z$. It is length vector. There is no change in z.

$x = s\cos\phi$
$y = s\sin\phi$
$Z = Z$



radius vector $\leftarrow \vec{s} = s\hat{s} \rightarrow$ unit vector

$\hat{s}$ will be +ve if move away from axis
" " −ve " " toward the axis.
$\hat{s}$ is the normal to the curved surface & +ve

Total Surface area $= \underbrace{\pi r^2}_{\text{cross sec.}} + \underbrace{2\pi r l}_{\text{cylentrical curved}}$

Limits :- $s : 0 \rightarrow \infty$
$\phi : 0 \rightarrow 2\pi$
$z : -\infty \rightarrow +\infty$

classmate
Date
Page

Length Elements :-

$\vec{dl_s} = ds\,\hat{s}$ dir^n of change $\rightarrow \hat{s}, \hat{\phi}, \hat{z}$

$\vec{dl_\phi} = s\,d\phi\,\hat{\phi}$

$\vec{dl_z} = dz\,\hat{z}$

Surface Element :-

dir^n of cylendrical curved surface is $\hat{s}$.

$\vec{dS_s} = \vec{dl_\phi} \times \vec{dl_z}$ $s \rightarrow \phi \rightarrow z \rightarrow s$

$= s\,d\phi\,dz\,(\hat{\phi} \times \hat{z})$ $\hat{x} \times \hat{y} = \hat{z}$

$\boxed{\vec{dS_s} = s\,d\phi\,dz\,\hat{s}}$ $\therefore$ $\hat{s} * \hat{\phi} = \hat{z}$

When flux pass through this curved surface we use this element. $\hat{\phi} \times \hat{z} = \hat{s}$, $\hat{z} \times \hat{s} = \hat{\phi}$

$s \rightarrow$ radius of cylender & remain const.

Dir^n of cross-sectional surface is $\hat{z}$ bcz it is normal to this surface.

$\vec{dS_z} = \vec{dl_s} \times \vec{dl_\phi}$

$= s\,ds\,d\phi\,\hat{z}$

If Press this cylender it'll become a rectangular sheet whose dir^n is $\hat{\phi}$ i.e. $\hat{\phi}$ is normal to this surface.

$\vec{dS_\phi} = \vec{dl_z} \times \vec{dl_s}$

$= ds\,dz\,\hat{\phi}$

Que:- Suppose a cylender of radius s & height h. Calculate its surface area.

$S_s = \int_S \vec{dS_s} = \int_S s\,d\phi\,dz$

$\int_S s\,ds\,d\phi = \frac{s^2}{2} \cdot 2\pi$

$$S_s = s\int_0^{2\pi} d\phi \int_{-h/2}^{h/2} dz = s[\phi]_0^{2\pi}[z]_{-h/2}^{h/2}$$

$$= s[2\pi]\left[\frac{h}{2}+\frac{h}{2}\right] = 2\pi s h$$

$$\boxed{S_s = 2\pi s h}$$

Q:- Calculate area of cross-sectional part.

$$S_z = \int_S s\, ds\, d\phi$$

$$= \int_0^s s\, ds \int_0^{2\pi} d\phi$$

$$= \left[\frac{s^2}{2}\right]_0^s [\phi]_0^{2\pi} = \frac{s^2}{2}\, 2\pi$$

$$S_z = \pi s^2$$

Volume

$$\cancel{S_\phi = \int_S ds\, dz}$$

$$\cancel{= \int_0^s ds \int}$$

Volume Element

$$d\tau = \vec{dl_s}\cdot(\vec{dl_\phi}\times\vec{dl_z})$$

$$\boxed{d\tau = s\, ds\, d\phi\, dz}$$

Relation b/w $\hat{s}, \hat{\phi}, \hat{z}$ & $\hat{x}, \hat{y}, \hat{z}$:-

$\hat{s} \rightarrow \hat{x}\ \hat{y}$

$\hat{\phi} \rightarrow \hat{x}\ \hat{y}$

$\hat{z} \rightarrow \hat{z}$

$$x = s\cos\phi \quad \text{——(1)}$$

$$y = s\sin\phi \quad \text{——(2)}$$

$$z = z \quad \text{——(3)}$$

$e_n(1)^2+(2)^2 \Rightarrow \quad x^2+y^2 = s^2(\cos^2\phi+\sin^2\phi)$

$$s^2 = x^2+y^2$$

$$\boxed{s = \sqrt{x^2+y^2}} \quad \text{——(4)}$$

Devide (2)/(1) $\Rightarrow \quad \tan\phi = \dfrac{y}{x}$

$$\boxed{\phi = \tan^{-1}\frac{y}{x}} \quad \text{——(5)}$$

& $\boxed{z = z}$ ——(6)

$\cancel{s = \sqrt{x^2}} \quad \vec{s} = x\hat{i}+y\hat{j}$

$$\boxed{\vec{s} = s\cos\phi\,\hat{i} + s\sin\phi\,\hat{j}} \quad \text{——(7)}$$

$\vec{s}$ is depanding on s & ϕ.

$$\hat{s} = \frac{\frac{\partial\vec{s}}{\partial s}}{\left|\frac{\partial\vec{s}}{\partial s}\right|}, \quad \hat{\phi} = \frac{\frac{\partial\vec{s}}{\partial\phi}}{\left|\frac{\partial\vec{s}}{\partial\phi}\right|},$$

$$\frac{\partial\vec{s}}{\partial s} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$$

$$\left|\frac{\partial\vec{s}}{\partial s}\right| = \sqrt{\cos^2\phi+\sin^2\phi} = 1$$

$$\boxed{\hat{s} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}} \quad \text{——(8)}$$

$$\frac{\partial\vec{s}}{\partial\phi} = -s\sin\phi\,\hat{i} + s\cos\phi\,\hat{j}$$

$$\left|\frac{\partial\vec{s}}{\partial\phi}\right| = \sqrt{s^2\sin^2\phi + s^2\cos^2\phi} = s\sqrt{(1)}$$

$$= s$$

$$\boxed{\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}} \quad \text{——(9)}$$

$$\boxed{\hat{z} = \hat{z}} \quad \text{——(10)}$$

(9) × $\cos\phi$ + (8) × $\sin\phi$ ⇒

$$\sin\phi\cos\phi\,\hat{i} + \sin^2\phi\,\hat{j} = \hat{s}\sin\phi$$

$$-\sin\phi\cos\phi\,\hat{i} + \cos^2\phi\,\hat{j} = \hat{\phi}\cos\phi$$

add $(\sin^2\phi + \cos^2\phi)\,\hat{j} = \hat{s}\sin\phi + \hat{\phi}\cos\phi$

$$\boxed{\hat{j} = \hat{y} = \sin\phi\,\hat{s} + \cos\phi\,\hat{\phi}} \quad \text{—— (11)}$$

eqⁿ (8) × $\cos\phi$ − (9) × $\sin\phi$

$$\cos^2\phi\,\hat{i} + \sin\phi\cos\phi\,\hat{j} = \cos\phi\,\hat{s}$$

$$-\sin^2\phi\,\hat{i} \pm \cos\phi\sin\phi\,\hat{j} = \sin\phi\,\hat{\phi}$$

sub. $\boxed{\hat{i} = \hat{x} = \cos\phi\,\hat{s} - \sin\phi\,\hat{\phi}}$ —— (12)

$$\boxed{\hat{z} = \hat{z}} \quad \text{—— (13)}$$

Gradient, Divergance & Curl :-

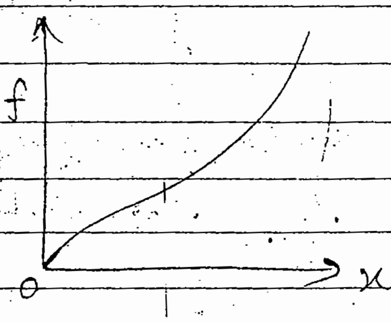
Gradient :- It is defined for a scalar. We can not take grad of any vector.

suppose a fucⁿ varry with x, $f(x)$

then $df = \underbrace{\left(\dfrac{\partial f}{\partial x}\right)}_{\text{slop}} dx$

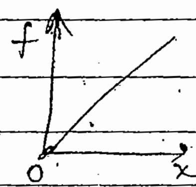
This derivative shows how f increases with x

This is the slop of the graph.



If graph linear then slop → const.

Non " → change



x → independent

f → dependent